Paper Reference(s)

# 6668/01

# Edexcel GCE

## Further Pure Mathematics FP2

## Advanced/Advanced Subsidiary

## Friday 6 June 2014 − Afternoon

## Time: 1 hour 30 minutes

| **Materials required for examination** | **Items included with question papers** |
|---|---|
| Mathematical Formulae (Pink) | Nil |

**Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.**

### Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

### Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

### Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

**P44512A**

**1.** (*a*) Express $\dfrac{2}{(r+2)(r+4)}$ in partial fractions.

**(1)**

(*b*) Hence show that

$$\sum_{r=1}^{n} \frac{2}{(r+2)(r+4)} = \frac{n(7n+25)}{12(n+3)(n+4)}$$

**(5)**

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**2.** Use algebra to find the set of values of $x$ for which

$$\left|3x^2 - 19x + 20\right| < 2x + 2$$

**(6)**

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**3.** $$y = \sqrt{8 + e^x}, \qquad x \in \sim$$

Find the series expansion for $y$ in ascending powers of $x$, up to and including the term in $x^2$, giving each coefficient in its simplest form.

**(8)**

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**4.** (*a*) Use de Moivre's theorem to show that

$$\cos 6\theta = 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1$$

**(5)**

(*b*) Hence solve for $0 \leq \theta \leq \dfrac{\pi}{2}$

$$64\cos^6\theta - 96\cos^4\theta + 36\cos^2\theta - 3 = 0$$

giving your answers as exact multiples of $\pi$.

**(5)**

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**5.** (*a*) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2}+2\frac{dy}{dx}+10y=27e^{-x}$$

**(6)**

(*b*) Find the particular solution that satisfies $y = 0$ and $\frac{dy}{dx} = 0$ when $x = 0$.

**(6)**

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**6.** The transformation $T$ from the $z$-plane, where $z = x + iy$, to the $w$-plane, where $w = u + iv$, is given by

$$w=\frac{4(1-i)z-8i}{2(-1+i)z-i}, \qquad z\neq\frac{1}{4}-\frac{1}{4}i$$

The transformation $T$ maps the points on the line $l$ with equation $y = x$ in the $z$-plane to a circle $C$ in the $w$-plane.

(*a*) Show that

$$w=\frac{ax^2+bxi+c}{16x^2+1}$$

where $a$, $b$ and $c$ are real constants to be found.

**(6)**

(*b*) Hence show that the circle $C$ has equation

$$(u-3)^2+v^2=k^2$$

where $k$ is a constant to be found.

**(4)**

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**7.** (*a*) Show that the substitution $v = y^{-3}$ transforms the differential equation

$$x\frac{dy}{dx}+y=2x^4y^4 \qquad \text{(I)}$$

into the differential equation

$$\frac{dv}{dx}-\frac{3v}{x}=-6x^3 \qquad \text{(II)}$$

**(5)**

(*b*) By solving differential equation (II), find a general solution of differential equation (I) in the form $y^3 = f(x)$.

**(6)**

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**8.**

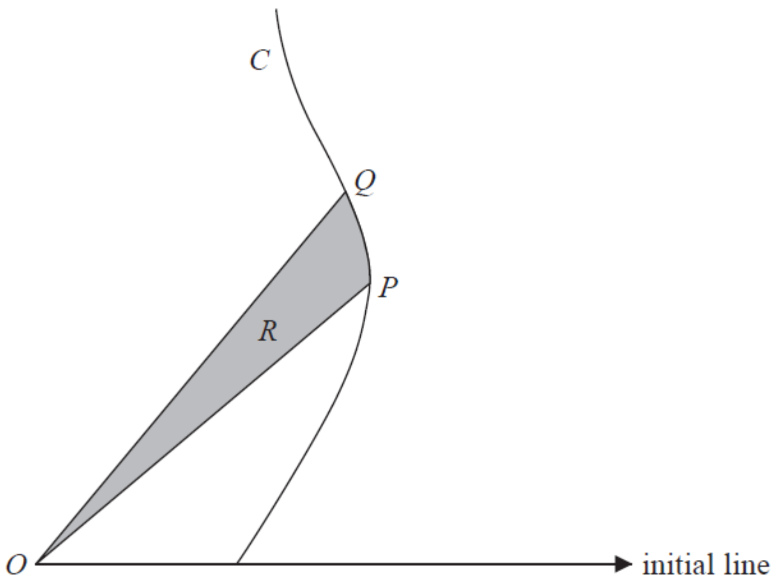


**Figure 1**

Figure 1 shows a sketch of part of the curve $C$ with polar equation

$$r = 1 + \tan\theta, \qquad 0 \leq \theta < \frac{\pi}{2}$$

The tangent to the curve $C$ at the point $P$ is perpendicular to the initial line.

($a$) Find the polar coordinates of the point $P$.

**(5)**

The point $Q$ lies on the curve $C$, where $\theta = \dfrac{\pi}{3}$.

The shaded region $R$ is bounded by $OP$, $OQ$ and the curve $C$, as shown in Figure 1.

($b$) Find the exact area of $R$, giving your answer in the form

$$\frac{1}{2}\left(\ln p + \sqrt{q} + r\right)$$

where $p$, $q$ and $r$ are integers to be found.

**(7)**

**TOTAL FOR PAPER: 75 MARKS**

**END**

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **1.(a)** | $\frac{2}{(r+2)(r+4)}=\frac{1}{r+2}-\frac{1}{r+4}$ | Correct partial fractions. Can be seen in (b) – give B1 for that. | B1 (1) |
| **(b)** | $\sum_{r=1}^{n}\frac{2}{(r+2)(r+4)}=\sum_{r=1}^{n}\left(\frac{1}{r+2}-\frac{1}{r+4}\right)$ | | |
| | $=\frac{1}{3}-\frac{1}{5}+\frac{1}{4}-\frac{1}{6}+........$ $+\frac{1}{n+1}-\frac{1}{n+3}+\frac{1}{n+2}-\frac{1}{n+4}$ | Attempts at least the first 2 terms and at least the last 2 terms as shown.(May be implied by later work) Must start at 1 and end at $n$ | M1 |
| | $=\frac{1}{3}+\frac{1}{4}-\frac{1}{n+3}-\frac{1}{n+4}$ | M1: Identifies their four fractions that do not cancel. If all terms are positive this mark is lost.<br>A1: Correct four fractions | M1A1 |
| | $=\frac{7}{12}-\frac{1}{n+3}-\frac{1}{n+4}$ | | |
| | $=\frac{7(n+3)(n+4)-12(n+4)-12(n+3)}{12(n+3)(n+4)}$ $=\frac{7n^2+49n+84-12n-48-12n-36}{12(n+3)(n+4)}$ | Attempt to combine at least 3 fractions, 2 of which have a function of $n$ in the denominator and expands the numerator. As a minimim, the product of 2 linear factors must be expanded in the numerator. | M1 |
| | $=\frac{n(7n+25)}{12(n+3)(n+4)}$ * | cso Must be factorised. If worked with $r$ instead of $n$ throughout, deduct last mark only. | A1 (5) |
| | | | **Total 6** |
| **(b) Way 2** | $\frac{7}{12}-\left(\frac{1}{n+3}+\frac{1}{n+4}\right)$ | | |
| | $=\frac{7}{12}-\left(\frac{n+4+n+3}{(n+3)(n+4)}\right)$ | | |
| | $=\frac{7(n+3)(n+4)-24n-84}{12(n+3)(n+4)}$ | | |
| | $=\frac{7n^2+49n+84-24n-84}{12(n+3)(n+4)}$ | Attempt to combine at least 3 fractions, 2 of which have a function of $n$ in the denominator and expands the numerator. Min as above | M1 |
| | $=\frac{n(7n+25)}{12(n+3)(n+4)}$ * | cso | A1 |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **2.** | $\left\|3x^2 - 19x + 20\right\| < 2x + 2$ | | |
| | $3x^2 - 19x + 20 = 2x + 2$ <br> $\Rightarrow 3x^2 - 21x + 18 = 0 \Rightarrow x = ..$ | $3x^2 - 19x + 20 = 2x + 2$ and attempt to solve correctly May be solved as an inequality | M1 |
| | $x = 1, \quad x = 6$ | Both (ie critical values seen) | A1 |
| | $-\left(3x^2 - 19x + 20\right) = 2x + 2$ <br> $\Rightarrow 3x^2 - 17x + 22 = 0 \Rightarrow x = ..$ | $-\left(3x^2 - 19x + 20\right) = 2x + 2$ and attempt to solve correctly May be solved as an inequality | M1 |
| | $x = \frac{11}{3}, \quad x = 2$ | Both (critical values seen) Accept awrt 3.67 | A1 |
| | $1 < x < 2, \quad \frac{11}{3} < x < 6$ | Must be strict inequalities. Accept awrt 3.67 A1 either correct, A1 both correct. But give A1A0 if both correct apart from $\leqslant$ seen somewhere in the final answers. <br> Give A1A0 if both correct and extra intervals seen | A1, A1 |
| | | | (6) |
| | | | **Total 6** |

If *no algebra* seen (implies a calculator solution) no marks.

*With algebra*:
M1 Squaring and reaching a quartic = 0
M1 Attempt to factorise and obtain at least one solution for $x$. Coefficient of $x^4$ and constant term correct for their quartic.
A1 Any 2 correct values
A1 All 4 correct values
Final 2 A marks as above

Accept set notation for the final 2 A marks. $x \in (1,2)$, $x \in \left(\frac{11}{3}, 6\right)$ not $[1,2]$

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **3.** | $y=\sqrt{8+e^x}$ | | |
| | $y=\left(8+e^x\right)^{\frac{1}{2}} \Rightarrow \frac{dy}{dx}=\frac{1}{2}\left(8+e^x\right)^{-\frac{1}{2}}\times e^x$ | M1: $\frac{dy}{dx}=k\left(8+e^x\right)^{-\frac{1}{2}}\times e^x$<br>A1: Correct differentiation | M1A1 |
| | $\frac{d^2y}{dx^2}=\frac{1}{2}\left(8+e^x\right)^{-\frac{1}{2}}\times e^x-\frac{1}{4}\left(8+e^x\right)^{-\frac{3}{2}}\times e^{2x}$ | M1: Correct use of the product rule $\frac{dy}{dx}=k\left(8+e^x\right)^{-\frac{1}{2}}\times e^x \pm K\left(8+e^x\right)^{-\frac{3}{2}}\times e^{(2)x}$<br>A1: Correct second derivative with $e^x\times e^x$ or $e^{2x}$ | M1A1 |
| | $f(0)=3$ | May only appear in the expansion | B1 |
| | $f'(0)=\frac{1}{6},\ f''(0)=\frac{17}{108}$ | Attempt both $f'(0)$ and $f''(0)$ with their derivatives found above | M1 |
| | $(y=)3+\frac{1}{6}x+\frac{17}{216}x^2$ | M1: Uses the correct Maclaurin series with their values. Accept 2 or 2! in $x^2$ term<br>A1: Correct expression | M1<br>A1cso<br>(8) |
| | | | **Total 8** |
| | **Alternative Methods:** | | |
| | $e^x=1+x+\frac{x^2}{2!}+\dots$ | 2 or 2! | |
| | $y=\left(9+x+\frac{x^2}{2}\dots\right)^{\frac{1}{2}}$ | M1: Subst corrrect expansion | M1 |
| | $=3\left(1+\frac{x}{9}+\frac{x^2}{9\times 2}+\dots\right)^{\frac{1}{2}}$ | B1: for 3<br>A1: for bracket | B1 A1 |
| | $=3\left(1+\frac{1}{2}\left(\frac{x}{9}+\frac{x^2}{2\times 9}\right)+\frac{\frac{1}{2}\times\left(-\frac{1}{2}\right)}{2!}\left(\frac{x}{9}+\frac{x^2}{2\times 9}\right)^2\right)$ | M1: Binomial expansion up to at least the squared term, 2 or 2! With squared term<br>A1: Correct expansion ie contents of bracket correct | M1A1 |
| | $=3+\frac{x}{6}+\frac{x^2}{12}-\frac{3}{8}\times\frac{x^2}{81}$ | M1 Remove all brackets | M1 |
| | $(y=)3+\frac{1}{6}x+\frac{17}{216}x^2$ | M1: Combine $x^2$ terms and obtain a 3 term quadratic<br>A1: Correct expression with or without $y=\dots$ | M1A1 |

**By implicit differentiation:** For the first 4 marks (rest as first method)

$$y^2=8+e^x$$

M1A1 $2y\frac{dy}{dx}=e^x$ M1A1 $2\frac{dy}{dx}+2y\frac{d^2y}{dx^2}=e^x$

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **4.(a)** | $\cos 6\theta = \text{Re}[(\cos\theta + i\sin\theta)^6]$ | Ignore any imaginary parts included in their expansion | |
| | $(\cos\theta + \text{i}\sin\theta)^6 = c^6 + 6c^5\text{i}s + 15c^4\text{i}^2s^2 + 20c^3\text{i}^3s^3 + 15c^2\text{i}^4s^4 + 6c\text{i}^5s^5 + \text{i}^6s^6$ | | M1 |
| | Attempt to expand correctly or only show real terms (May be implied)<br>Often seen with powers of i simplified.<br>If $\text{i}s^n$ seen, but becomes $\text{i}^ns^n$ (oe) later, deduct the final A mark of (a) even if no further errors. | | |
| | $\cos 6\theta = c^6 - 15c^4s^2 + 15c^2s^4 - s^6$ | M1: Attempt to identify real parts. These 2 M marks may be awarded together<br>A1: Correct expression | M1A1 |
| | $= c^6 - 15c^4(1-c^2) + 15c^2(1-c^2)^2 - (1-c^2)^3$ | | M1 |
| | Correct use of $s^2 = 1 - c^2$ in all their sine terms | | |
| | $\cos 6\theta = c^6 - 15c^4 + 15c^6 + 15c^2(1 - 2c^2 + c^4) - (1 - 3c^2 + 3c^4 - c^6)$ | | |
| | $\cos 6\theta = 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1$ * | ( $\cos 6\theta$ must be seen somewhere) | A1cso |
| | | | (5) |
| **(b)** | $64\cos^6\theta - 96\cos^4\theta + 36\cos^2\theta - 3 = 0$<br>$\Rightarrow 2\cos 6\theta - 1 = 0 \therefore \underline{\cos 6\theta = \tfrac{1}{2} \text{ or } 0.5}$ | M1: Uses part (a) to obtain an equation in $\cos 6\theta$<br>A1: Correct underlined equation | M1A1 |
| | $\cos 6\theta = \tfrac{1}{2} \Rightarrow (6\theta =)\ \dfrac{\pi}{3}, \dfrac{5\pi}{3}, \dfrac{7\pi}{3}$ | | |
| | $\theta = \dfrac{\pi}{18}, \dfrac{5\pi}{18}, \dfrac{7\pi}{18}$ | M1: Valid attempt to solve $\cos 6\theta = k,\ -1 \le k \le 1$ leading to $\theta = \ldots$ Can be degrees<br>A1 2 correct answers<br>A1 3rd correct answer with no extras within the range, ignore extras outside the range. Must be radians<br>Answers in degrees or decimal answers score A0A0 | M1<br>A1A1 |
| | | | (5) |
| | | | **Total 10** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **5.(a)** | $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+10y=27e^{-x}$ | | |
| | $m^2+2m+10\ (=0)\Rightarrow m=...$ | Form and solve the aux equation | M1 |
| | $m=-1\pm3i$ | | A1 |
| | $(y=)e^{-x}(A\cos3x+B\sin3x)$ or $(y=)Ae^{(-1+3i)x}+Be^{(-1-3i)x}$ | $y=$ not needed May be seen with $\theta$ instead of $x$ | A1 |
| | $y=ke^{-x},\ y'=-ke^{-x},\ y''=ke^{-x}$ | $y=ke^{-x}$ and attempt to differentiate twice | M1 |
| | $e^{-x}(k-2k+10k)=27e^{-x}\Rightarrow k=3$ | | A1 |
| | $y=e^{-x}(A\cos3x+B\sin3x+3)$ or $y=Ae^{(-1+3i)x}+Be^{(-1-3i)x}+3e^{-x}$ | Must be $x$ and have $y$ = ....Ignore any attempts to change the second form. (But see note at end about marking (b)) ft, so $y$ = their CF + their PI | B1ft (NB A1 on e-PEN) |
| | | | (6) |
| **(b)** | $x=0, y=0\Rightarrow A=(-3)$ | Uses $x=0, y=0$ in an attempt to find $A$ | M1 |
| | $y'=-e^{-x}(A\cos3x+B\sin3x+3)+e^{-x}(3B\cos3x-3A\sin3x)$ | M1: Attempt to differentiate using the product rule, with $A$ or their value of $A$<br>A1: Correct derivative, with $A$ or their value of $A$ | M1A1 |
| | $x=0, y'=0\Rightarrow B=0$ | M1: Uses $x=0, y'=0$ and their value of $A$ in an attempt to find $B$<br>A1: $B$ = 0 | M1A1 |
| | $y=e^{-x}(3-3\cos3x)$ oe | cao and cso | A1 (6) |
| | | | **Total 12** |
| | **Alternative for (b) using** | $y=Ae^{(-1+3i)x}+Be^{(-1-3i)x}+3e^{-x}$ | |
| | $x=0, y=0$ to get an equation in $A$ and $B$ | May come from the real part of their derivative instead | M1 |
| | $y'=(-1+3i)Ae^{(-1+3i)x}+(-1-3i)Be^{(-1-3i)x}-3e^{-x}$ | M1: Attempt differentiation using chain rule<br>A1: Correct differentiation | M1A1 |
| | $x=0, y'=0\Rightarrow -A-B-3=0$ from real parts and $3A-3B=0$ from imaginary parts<br>So $A=B=-\frac{3}{2}$ | M1: Uses $x=0, y'=0$ and equates imaginary parts to obtain a second equation for $A$ and $B$ and attempts to solve their equations<br>A1: $A=B=-\frac{3}{2}$ | M1A1 |
| | $y=-\frac{3}{2}e^{(-1+3i)x}-\frac{3}{2}e^{(-1-3i)x}+3e^{-x}$ | A1: Ignore any attempts to change. | A1 |

Some may change the second form in (a) before proceeding to (b). If their changed form is correct, all marks for (b) are available; if their changed form is incorrect only M marks are available.

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **6.(a)** | $w=\dfrac{4(1-i)z-8i}{2(i-1)z-i}$ | | |
| | | | |
| | **Method 1………:Substituting** $z=x+xi$ | **at the start** | |
| | $w=\dfrac{4(1-i)(x+xi)-8i}{2(i-1)(x+xi)-i}$ | Substitutes for $z$ | M1 |
| | $w=\dfrac{4(x+xi-xi+x)-8i}{2(xi-x-x-xi)-i}$ | M1: Attempt to expand numerator and denominator<br>A1: Correct expression | M1A1 |
| | $\dfrac{8i-8x}{4x+i}.\dfrac{4x-i}{4x-i}$ | M1: Multiplies numerator and denominator by the conjugate of their denom. No expansion needed<br>A1: Uses correct conjugate (not ft) | M1A1 |
| | $=\dfrac{-32x^2+40xi+8}{16x^2+1}$ | cso Award only if final answer is correct and follows correct working | B1 |
| | **NB:** The B mark appears first on e-PEN but will be awarded last | | |
| | | | (6) |
| | | | |
| | **Method 2: if they proceed without *y* = *x*** (substitution may happen anywhere in the working) | | |
| | $w=\dfrac{(1-i)z-8\mathrm{i}}{2(-1+\mathrm{i})-\mathrm{i}}=\dfrac{4(1-\mathrm{i})(x+\mathrm{i}y)-8\mathrm{i}}{2(-1+\mathrm{i})(x+\mathrm{i}y)-\mathrm{i}}$ | Substitutes for $z$ | M1 |
| | $=\dfrac{4(1-\mathrm{i})x+4(1-\mathrm{i})\mathrm{i}y-8\mathrm{i}}{2(-1+\mathrm{i})x+2(-1+\mathrm{i})\mathrm{i}y-\mathrm{i}}$ | Attempt to expand numerator and denominator | M1 |
| | $=\dfrac{4x+4y+(4y-4x-8)\mathrm{i}}{-2x-2y+(2x-2y-1)\mathrm{i}}$ | Correct expression | A1 |
| | $=\dfrac{4x+4y+(4y-4x-8)\mathrm{i}}{-2x-2y+(2x-2y-1)\mathrm{i}}\times\dfrac{-2x-2y-(2x-2y-1)\mathrm{i}}{-2x-2y-(2x-2y-1)\mathrm{i}}$<br>M1: Multiplies numerator and denominator by the conjugate of their denom. No expansion needed.<br>A1: Uses correct conjugate. (not ft) | | M1A1 |
| | $=\dfrac{-16x^2-16y^2+12y-12x+8+(20x+20y)\mathrm{i}}{8x^2+8y^2-4x+4y+1}$ | | |
| | $=\dfrac{-32x^2+40x\mathrm{i}+8}{16x^2+1}$ | cso Correct answer using $y=x$<br>Award only if final answer is correct and follows correct working | B1 |
| | **NB:** The B mark appears first on e-PEN but will be awarded last | | (6) |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **NB:** The order of awarding the marks here has changed from the original mark scheme, but they must still be entered on e-PEN by their descriptors (M or A) | | | |
| **6(b)** | $u=\frac{-32x^2+8}{16x^2+1},\ \ v=\frac{40x}{16x^2+1}$ | Identifies *u* and *v* (Real and imaginary parts)<br>May be implied by their working and may be in terms of x and y. | M1<br><br>1st M mark on e-PEN |
| | $\left(\frac{8-32x^2}{16x^2+1}-3\right)^2+\left(\frac{40x}{16x^2+1}\right)^2$ | Substitutes for their *u* and *v* in the given equation. May be in terms of *x* and *y*. May have *a*, *b*, *c* instead of their values (which may be chosen by the candidate if unable to do (a)) | dM1<br><br>2nd M mark on e-PEN |
| | $=\left(\frac{8-32x^2-48x^2-3}{16x^2+1}\right)^2+\left(\frac{40x}{16x^2+1}\right)^2$ | | |
| | $=\frac{\left(5-80x^2\right)^2}{\left(16x^2+1\right)^2}+\frac{1600x^2}{\left(16x^2+1\right)^2}$ | | |
| | $=\frac{6400x^4+800x^2+25}{\left(16x^2+1\right)^2}$ | Combines to form a single correct fraction | A1<br><br>1st A mark on e-PEN |
| | $=\frac{25\left(16x^2+1\right)^2}{\left(16x^2+1\right)^2}=25$ | $k=5$ or $k^2=25$ may (but need not) be seen explicitly | A1<br><br>2nd A mark on e-PEN |
| | | | |
| | | | (4) |
| | | | **Total 10** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| | **Way 1** | | |
| **7.(a)** | $v = y^{-3} \Rightarrow \dfrac{\mathrm{d}v}{\mathrm{d}y} = -3y^{-4}$ | Correct derivative | B1 |
| | $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{d}y}{\mathrm{d}v}\dfrac{\mathrm{d}v}{\mathrm{d}x} = -\dfrac{y^4}{3}\dfrac{\mathrm{d}v}{\mathrm{d}x}$<br>Or $-3y^{-4}\dfrac{\mathrm{d}y}{\mathrm{d}x}x - 3y^{-3} = -6x^4$ | M1: Correct use of the chain rule<br>A1: Correct equation | M1A1 |
| | $-\dfrac{y^4}{3}\dfrac{\mathrm{d}v}{\mathrm{d}x}x + y = 2x^4y^4$ | | |
| | $-\dfrac{y^4}{3}\dfrac{\mathrm{d}v}{\mathrm{d}x}x + y = 2x^4y^4 \Rightarrow \dfrac{\mathrm{d}v}{\mathrm{d}x} - \dfrac{3v}{x} = -6x^3$ | dM1: Substitutes to obtain an equation in $v$ and $x$.<br>A1: Correct completion with no errors seen | dM1A1 |
| | | | |
| | **Way 2** | | |
| | $y = v^{-\frac{1}{3}} \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}v} = -\frac{1}{3}v^{-\frac{4}{3}}$ | Correct derivative | B1 |
| | $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{d}y}{\mathrm{d}v}\dfrac{\mathrm{d}v}{\mathrm{d}x} = -\frac{1}{3}v^{-\frac{4}{3}}\dfrac{\mathrm{d}v}{\mathrm{d}x}$ | M1: Correct use of the chain rule<br>A1: Correct equation | M1A1 |
| | $-\dfrac{v^{-\frac{4}{3}}}{3}\dfrac{\mathrm{d}v}{\mathrm{d}x}x + v^{-\frac{1}{3}} = 2x^4v^{-\frac{4}{3}}$ | dM1: Substitutes to obtain an equation in $v$ and $x$. | dM1 |
| | $-\dfrac{v^{-\frac{4}{3}}}{3}\dfrac{\mathrm{d}v}{\mathrm{d}x}x + v^{-\frac{1}{3}} = 2x^4v^{-\frac{4}{3}} \Rightarrow \dfrac{\mathrm{d}v}{\mathrm{d}x} - \dfrac{3v}{x} = -6x^3$ | A1: Correct completion with no errors seen | A1 |
| | | | |
| | **Way 3 (Working in reverse)** | | |
| | $v = y^{-3} \Rightarrow \dfrac{\mathrm{d}v}{\mathrm{d}y} = -3y^{-4}$ | B1: Correct derivative | B1 |
| | $\dfrac{\mathrm{d}v}{\mathrm{d}x} = \dfrac{\mathrm{d}v}{\mathrm{d}y}\dfrac{\mathrm{d}y}{\mathrm{d}x} = -3y^{-4}\dfrac{\mathrm{d}y}{\mathrm{d}x}$ | M1: Correct use of chain rule<br>A1: Correct expression for d$v$/d$x$ | M1A1 |
| | $-3y^{-4}\dfrac{\mathrm{d}y}{\mathrm{d}x} - \dfrac{3y^{-3}}{x} = -6x^3$ | M1: Substitutes correctly for $\dfrac{\mathrm{d}v}{\mathrm{d}x}$ and $v$ in equation (II) to obtain a D.E. in terms of $x$ and $y$ only.<br>A1: Correct completion to obtain equation (I) with no errors seen | dM1A1 |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **7(b)** | $I = e^{\int -\frac{3}{x}dx} = e^{-3\ln x} = \frac{1}{x^3}$ | M1: $e^{\int \pm\frac{3}{x}dx}$ and attempt integration. If not correct, ln $x$ must be seen. | M1A1 |
| | | A1: $\frac{1}{x^3}$ | |
| | $\frac{v}{x^3} = \int -6\,dx = -6x(+c)$ | M1: $v \times$ their $I = \int -6x^3 \times$ their $I$ d$x$ | dM1A1 |
| | | A1: Correct equation with or without + $c$ | |
| | $\frac{1}{y^3x^3} = -6x + c \Rightarrow y^3 = ....$ | Include the constant, then substitute for $y$ and attempt to rearrange to $y^3$ =.... or $y$ = .... with the constant treated correctly | ddM1 dep on both M marks of (b) |
| | $y^3 = \frac{1}{cx^3 - 6x^4}$ | Or equivalent | A1 (6)<br>**Total 11** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| | $r = 1 + \tan\theta$ | | |
| **8.(a)** | $x = r\cos\theta \Rightarrow x = (1+\tan\theta)\cos\theta$ | States or implies $x = r\cos\theta$ | M1 |
| | $x = \cos\theta + \sin\theta,\ \dfrac{\mathrm{d}x}{\mathrm{d}\theta} = \cos\theta - \sin\theta$ | M1: Attempt to differentiate $x = r\cos\theta$ or $x = r\sin\theta$<br>A1: Correct derivative | M1A1 |
| Alt for the 2 diff marks | $\dfrac{\mathrm{d}x}{\mathrm{d}\theta} = \sec^2\theta\cos\theta + (1+\tan\theta)(-\sin\theta)$ | M1: Attempt to differentiate using product rule (dep on first M1)<br>A1: correct (unsimplified) differentiation | |
| | $\dfrac{\mathrm{d}x}{\mathrm{d}\theta} = 0 \Rightarrow \tan\theta = 1 \Rightarrow \theta = ....$ | Set their derivative = 0 and attempt to solve for $\theta$<br>(Dependent on second M mark above) | dM1 |
| | $\theta = \dfrac{\pi}{4},\ r = 2$ | Both | A1 |
| | **NB:** Use of $x = r\sin\theta$ can score M0M1A0M1A0 max | | (5) |
| | | | |
| **(b)** | $\int r^2\mathrm{d}\theta = \int (1+\tan\theta)^2\mathrm{d}\theta$ | Use of $\int r^2\mathrm{d}\theta$ and $r = 1+\tan\theta$ No limits needed | M1 |
| | $(1+\tan\theta)^2 = 1 + 2\tan\theta + \tan^2\theta$<br>$= 1 + 2\tan\theta + \sec^2\theta - 1$ | Expands and uses the correct identity | M1 |
| | $\int (2\tan\theta + \sec^2\theta)\mathrm{d}\theta$ | Correct expression Need not be simplified, no limits needed. | A1 |
| | $\left[2\ln\sec\theta + \tan\theta\right]_{\left(\frac{\pi}{4}\right)}^{\left(\frac{\pi}{3}\right)}$ | M1: Attempt to integrate – at least one trig term integrated. Dependent on the second M mark<br>A1: Correct integration. Need not be simplified or include limits. | dM1A1 |
| | $R = \dfrac{1}{2}\left\{\left(2\ln\sec\dfrac{\pi}{3} + \tan\dfrac{\pi}{3}\right) - \left(2\ln\sec\dfrac{\pi}{4} + \tan\dfrac{\pi}{4}\right)\right\}$ | Substitutes $\dfrac{\pi}{3}$ and their $\dfrac{\pi}{4}$ and subtracts<br>(Dependent on 2 previous method marks in (b)) | dM1 |
| | $R = \dfrac{1}{2}\left\{\ln 2 + \sqrt{3} - 1\right\}$ | Cao and cso | A1 |
| | | | (7) |
| | | | **Total 12** |